Time: 3 hrs   Date: -01-07
                                        Instructor: Pl Muthuramalingam

Note: <u>Maximum marks you can get is 40 out of 41.</u>

1. Let $V = \{(x, y, z) : \quad r^2 \leq x^2 + y^2 + z^2 \leq 1, \ z \geq 0, y \geq 0, x \geq 0\}$ where $0 < r < 1$ and S be the boundary of V. Describe the surface S with outernormal orientation.   [10]

2. State Stokes theorem and Divergence theorem.   [4]

3. Prove Weirstrass theorem for the interval $[0, 1]$.   [10]

4. Let S be nonempty set. $f_1, f_2, \ldots : S \to R$ are bounded functions and $f_n \to g$ uniformly on S for a suitable function g. Show that $\sup\limits_{n} \sup\limits_{x \in S} |f_n(x)|$ is finite.   [5]

5. Let $\phi : R^2 \to R$ be the function $\phi(x, y) = x^2 + y^2 - 1$, so that $\phi(1, 0) = 0$. Show that there does not exist a continuously differentiable function $g : (1 - \delta, 1 + \delta) \to R$ such that $\phi(t, \ g(t)) = 0$, with $g(1) = 0$.   [3]

6. Let $a, b > 0 \quad c < 0$. Assume that $ax^2 + 2hxy + by^2 + c = 0$ represents an ellipse. Find the area enclosed by it in terms of a, b, c, h.   [4]

7. Let $f : R^3 \to R$ be a smooth scalar field and $K : R^3 \to R^3$ a smooth vector field. Show that $\mathrm{div}(f\underset{\sim}{K}) = f \ \mathrm{div}\underset{\sim}{K} + (\tilde{\nabla} f) \cdot \underset{\sim}{K}$.   [3]

8. Let $f_n(x) = \frac{1}{1+nx}$ for $x \geq 0 \quad n = 1, 2, 3, \ldots$. Show that f_n does not converge uniformly on $[0, \delta]$ for any $\delta > 0$.   [2]